The Thirteenth International Symposium on Artificial Life and Robotics 2008(AROB 13th '08),
B-Con Plaza, Beppu, Oita, Japan, January 31-February 2, 2008

Path-Bounded Three-Dimensional Finite Automata

Makoto SAKAMOTO[1], Masatsugu FUKUDA[1], Satoshi OKATANI[1], Takao ITO[1], Hiroshi FURUTANI[1], and Michio KONO[1]

[1] Dept. of Computer Science and Systems Engineering, University of Miyazaki, Miyazaki 889-2192, JAPAN

Abstract

The comparative study of the computational powers of deterministic and nondeterministic computations is one of the central tasks of complexity theory. This paper investigates the computational power of nondeterministic computing devices with restricted nondeterminism. There are only few results measuring the computational power of restricted nondeterminism. In general, there are three possibilities to measure the amount of nondeterminism in computation. In this paper, we consider the possibility to count the number of different nondeterministic computation paths on any input. In particular, we deal with five-way three-dimensional finite automata with multiple input heads operating on three-dimensional input tapes.

Key Words **:** computational complexity, finite automaton, multihead, path-bounded, three-dimension.

1 Introduction

The question of whether processing three-dimensional digital patterns is much difficult than two-dimensional ones is of great interest from the theoretical and practical standpoints. In recent years, due to the advances in many application areas such as computer graphics, computer-aided design / manufacturing, computer vision, image processing, robotics, and so on, the study of three-dimensional pattern processing has been of crucial importance. Thus, the study of three-dimensional automata as the computational model of three-dimensional pattern processing has been meaningful. For example, in [8,9], a three-dimensional finite automaton was proposed as a natural extension of the two-dimensional finite automaton to three dimensions. On the other hand, the comparative study of the computational powers of deterministic computations is one of the central tasks of complexity theory.

In this paper, we investigate the computational power of nondeterministic computing devices with restricted nondeterminism. However, there are only few results [1-4] measuring the computational power of restricted nondeterminism. In general, there are three possibilities to measure the amount of nondeterminism in computation. One possibility is to count the number of advice bits (nondeterministic guesses) in particular nondeterministic computations, and the second possibility is to count the number of accepting computation paths. The third possibility is to count the number of different nondeterministic computation paths on any input. This paper considers the third one. In particular, the paper investigates a hierarchy on the degree of nondeterminism of five-way three-dimensional (simple) multi-head finite automata as a natural extension of the three-way two-dimensional (simple) multi-head finite automata [5]. Furthermore, we investigate a relationship between the accepting powers of nondeterminism and self-verifying nondeterminism for five-way three-dimensional (simple) multi-head finite automata with the number of computation paths restricted (see Fig.1).

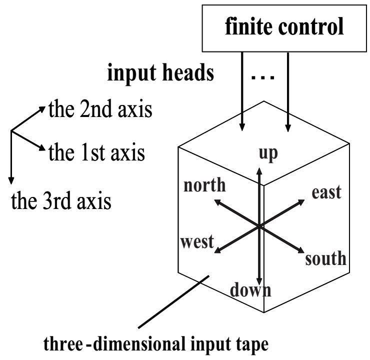

Fig. 1: Three-dimensional multihead finite automaton.

2 Preliminaries

Let $\sum$ be a finite set of symbols. A *three-dimensional tape* over $\sum$ is a three-dimensional rectangular array of elements of $\sum$. The set of all three-dimensional tapes over $\sum$ is denoted by $\sum^{(3)}$. Given a tape $x \in \sum^{(3)}$, for each integer j ($1 \leq j \leq 3$), we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \sum^{(3)}$ with $l_1(x) = n_1, l_2(x) = n_2$, and $l_3(x) = n_3$ is denoted by $\sum^{(n_1\ n_2\ n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each $j (1 \leq j \leq 3)$, let $x(i_1, i_2, i_3)$ denote the symbol in x with coordinates (i_1, i_2, i_3). Furthermore, we define

$$x[(i_1, i_2, i_3), (i'_1, i'_2, i'_3)],$$

when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j (1 \leq j \leq 3)$, as the three-dimensional input tape y satisfying the following conditions :

(i)for each $j (1 \leq j \leq 3)$, $l_j(y) = i'_j - i_j + 1$;

(ii)for each $r_1, r_2, r_3 (i \leq r_1 \leq l_1(y), \; i \leq r_2 \leq l_2(y), \; i \leq r_3 \leq l_3(y))$, $y(r_1, r_2, r_3) = x(r_1 + i_1 - 1, \; r_2 + i_2 - 1, \; r_3 + i_3 - 1)$. (We call $x[(i_1, i_2, i_3), (i'_1, i'_2, i'_3)]$ the $[(i_1, i_2, i_3), (i'_1, i'_2, i'_3)]$-*segment* of x.)

For each $x \in \sum^{(n_1 \; n_2 \; n_3)}$ and for each $1 \leq i_1 \leq n_1$, $1 \leq i_2 \leq n_2$, $1 \leq i_3 \leq n_3$, $x[(i_1, 1, 1), (i_1, n_2, n_3)$, $x[(1, i_2, 1), (n_1, i_2, n_3)]$, $x[(1, 1, i_3), (n_1, n_2, i_3)]$, $x[(i_1, 1, i_3), (i_1, n_2, i_3)]$, and $x[(1, i_2, i_3), (n_1, i_2, i_3)]$ are called the i_1*th* (2-3) *plane of* x, the i_2*th* (1-3) *plane of* x, the i_3th (1-2) *plane of* x, the i_1*th* now *on the* i_3*th* (1-2) *plane of* x, and the i_2*th column on* the i_3*th* (1-2) *plane of* x, and are denoted by $x(2\text{-}3)_{i_1}$, $x(1\text{-}3)_{i_2}$, $x(1\text{-}2)_{i_3}$, $x[i_1, *, i_3]$, and $x[*, i_2, i_3]$, respectively.

A *five-way three-dimensional multihead finite automaton* (*FV3-MHFA*) [5] is a finite automaton with multiple input heads operating on three-dimensional input tapes surrounded by boundary symbols #'s. These heads can move east, west, south, north, or down but not up. A *five-way three-dimensional simple multihead finite automaton* (*FV3-SMHFA*) is an *FV3-MHFA* which has only one reading head and other counting heads which can only detect whether they are on the boundary symbols or a symbol in the input alphabet.

When a three-dimensional input tape x is presented to a three-dimensional device M, M starts in its initial state with all its heads on $x(1, 1, 1)$. M *accepts* the input tape x if and only if it eventually halts in an accepting state with all its heads on the bottom boundary symbols #'s.

For a device M, we denote by $T(M)$ the set of all inputs accepted by M. The states of this device are considered to be divided into three disjoint sets of working, accepting, and rejecting states.

A *self-verifying nondeterministic* device is a device with four types of states : working, accepting, rejecting, and neutral ones. The self-verifying nondeterministic device M is not allowed to make mistakes. If there is a computation of M on an input x finishing in an accepting (resp., rejecting) state, then x must be in $T(M)$ (resp., x must not be in $T(M)$). For every input y, there is at least one computation of M that finishes either in an accepting state (if $y \in T(M)$) or in a rejecting state (if $y \notin T(M)$).

For each $k \geq 1$, let *FV3-kHFA* denote a *five-way three-dimensional k-head finite automaton*. In order to represent different kinds of *FV3-kHFA*'s, we use the notation *FV3-XYkHFA*, where

$$(1) \left\{ \begin{array}{l} X = N : nondeterministic, \\ X = SVN : self-verifying \; nondeterministic \; ; \end{array} \right.$$

$$(2) \left\{ \begin{array}{l} Y = SP : simple, \\ there \; is \; no \; Y \; : \; non-simple. \end{array} \right.$$

We denote by $\mathcal{L}[FV3\text{-}XYkHFA]$ the class of sets of input tapes accepted by *FV3-XYkHFA*'s.

Let r be a positive integer. A device M described above is r *path-bounded* if for any input x, there are at most r computation paths of M on x. We denote an r path-bounded *FV3-XYkHFA* by *FV3-XYkHFA*(r), and denote the class of sets of input tapes accepted by *FV3-XYkHFA*(r)'s by $\mathcal{L}[FV3\text{-}XYkHFA(r)]$.

3 Non-Simple Case

We first prove a strong separation between r path-bounded and $(r+1)$ path-bounded for five-way three-dimensional multihead finite automata.

Theorem 3.1. *For each positive integers* $k \geq 2$ *and* $r \geq 1$,

$$\mathcal{L}[FV3\text{-}SVNkHFA(r+1)] - \mathcal{L}[FV3\text{-}NkHFA(r)] \neq \phi.$$

Proof : For each positive integers $k \geq 2$ and $r \geq 1$, let $T_1(k, r) = \{x \in \{0,1\}^{(3)} \mid {}^{\exists} n \geq 2rb(k)+1 \; [l_1(x) = l_2(x) = l_3(x) = n] \wedge {}^{\exists} i (0 \leq i \leq r\text{-}1) \; [{}^{\forall} j (ib(k)+1 \leq j \leq (i+1)b(k)) \; [x[*, *, j] = x[*, *, 2rb(k)\text{-}j+1] \wedge {}^{\exists} z \in \{0,1\} \; [x[*, *, 2rb(k)+1] = 0^i 1 z$ (the string of the symbols forms a line from the first column to the last column in the $(2rb(k)+1)$th plane and from the first row to the last row in a column one after another)$]]\}$, where $b(k) = {}_k C_2$. To prove the theorem, it suffices to show that for each $k \geq 2$ and $r \geq 1$, (1) $T_1(k, r+1) \in \mathcal{L}[FV3\text{-}SVNkHFA(r+1)]$, and (2) $T_1(k, r+1) \notin \mathcal{L}[FV3\text{-}NkHFA(r)]$. First of all we prove Past (1) of the theorem. $T_1(k, r+1)$ is accepted by an *FV3-SVNkHFA*$(r+1)$ M which acts as follows. Suppose that an input tape x with $l_1(x) = l_2(x) = l_3(x) = n$ $(n \geq 2(r+1)b(k)+1)$ is presented to M. First, M nondeterministically guesses some i $(0 \leq i \leq r)$ and checks whether $x[*, *, j]$ and $x[*, *, 2(r+1)b(k)\text{-}j+1]$ are identical for each j $(ib(k)+1 \leq j \leq (i+1)b(k))$. This check can easily be done by using a well-known technique in [10]. If $x[*, *, j] \neq x[*, *, 2(r+1)b(k)\text{-}j+1]$ for some j $(ib(k)+1 \leq j \leq (i+1)b(k))$ and $x[*, *, 2(r+1)b(k)+1] = 0^i 1 z$ (the string of the symbols forms a line from the first column to the last column in the $(2(r+1)b(k)+1)$th place and from the first row to the last row in a column one after another) for some $z \in \{0, 1\}$, then M enters a rejecting state. If $x[*, *, 2(r+1)b(k)+1] \neq 0^i 1 z$ (the string of the symbols forms a line from the first column to the last column in the $(2(r+1)b(k)+1)$th place and from the first row to the last row in a column one after another) for some $z \in \{0, 1\}$, M enters a neutral state, whether or not $x[*, *, j] = x[*, *, 2(r+1)b(k)\text{-}j+1]$ for each j $(ib(k)+1 \leq j \leq (i+1)b(k))$. It is obvious that M accepts $T_1(k, r+1)$. On the other hand, by using a standard technique in [6, 7], we can get Part (2) of the theorem.

From Theorem 3.1, we have the following corollary :

Corollary 3.1. *For each* $X \in \{N, SVN\}$, *and for each positive integers* $k \geq 2$ *and* $r \geq 1$,

$$\mathcal{L}[FV3\text{-}XkHFA(r)] \subsetneq \mathcal{L}[FV3\text{-}XkHFA(r{+}1)].$$

We next show a strong separation between self-verifying nondeterminism and nondeterminism.

Theorem 3.2. *For each positive integer* $k \geq 2$.

$$\mathcal{L}[FV3\text{-}NkHFA(2)] - \mathcal{L}[FV3\text{-}SVNkHFA] \neq \phi.$$

Proof : For each positive integer $k \geq 2$, let $T_2(k) = \{x \in \{0,1\}^{(3)} \mid \exists n \geq 4b(k)\ [l_1(x) = l_2(x) = l_3(x) = n] \wedge \exists i\,(0 \leq i \leq 1)\ \exists j\,(ib(k)+1 \leq j \leq (i+1)b(k))\ [x[*,*,j] \neq x[*,*,4b(k)-j+1]\}$, where $b(k) = {}_kC_2$. Then, we have $T_2(k) \in \mathcal{L}[FV3\text{-}NkHFA(2)] - \mathcal{L}[FV3\text{-}SVNkHFA]$. Then, by using the same idea as in [6,7], we can get the desired result.

From Theorems 3.1 and 3.2, we have the following corollary:

Corollary 3.2. *For each positive integers* $k \geq 2$ *and* $r \geq 2$,

(1) $\mathcal{L}[FV3\text{-}SVNkHFA] \subsetneq \mathcal{L}[FV3\text{-}NkHFA]$,

(2) $\mathcal{L}[FV3\text{-}SVNkHFA(r)] \subsetneq \mathcal{L}[FV3\text{-}NkHFA(r)]$, *and*

(3) $\mathcal{L}[FV3\text{-}SVNkHFA(r{+}1)]$ *and* $\mathcal{L}[FV3\text{-}NkHFA(r)]$ *are incomparable.*

4 Simple Case

This section first prove a strong separation between r path-bounded and $(r+1)$ path-bounded machines for the five-way simple case.

Theorem 4.1. *For each positive integers* $k \geq 2$ *and* $r \geq 1$,

$$\mathcal{L}[FV3\text{-}SVNSPkHFA(r{+}1)] - \mathcal{L}[FV3\text{-}NSPkHFA(r)] = \phi.$$

proof : For each positive integers $k \geq 2$ and $r \geq 1$, let $T_3(k, r) = \{x \in \{0, 1\}^{(3)} \mid \exists n \geq \max\{2r+1, k\}$ $[l_1(x) = l_2(x) = l_3(x) = n] \wedge [\,[$(the 1st plane of x has exactly k '1's) $\wedge\ x[*, *, 1] = x[*, *, 1+r] \wedge \exists z \in \{0, 1\}$ $[x[*, *, 2r+1] = 01z$ (the string of the symbols forms a line from the first column to the last column in the $(2r+1)$th plane and from the first row to the last row in a column one after another)]] $\vee$ [(the 2nd plane of x has exactly k '1's) $\wedge\ x[*, *, 2] = x[*, *, 2+r] \wedge \exists z \in \{0, 1\}$ $[x[*, *, 2r+1] = 0^2 1z$ (the string of the symbols forms a line from the first column to the last column in the $(2r+1)$th plane and from the first row to the last row in a column one after another)]] $\vee \cdots \vee$ [(the rth plane of x has exactly k '1's) $\wedge\ x[*, *, r] = x[*, *, 2r] \wedge \exists z \in \{0, 1\}$ $[x[*, *, 2r+1] = 0^r 1z$ (the string of the symbols forms a line from the first column to the last column in the $(2r+1)$th plane and from the first row to the last row in a column one after another)]]]$\}$. By using the same technique as in the proof of Theorem 4.1 in [7], we can get the desired result.

From Theorem 4.1, we have the following corollary :

Corollary 4.1. *For each* $X \in \{N, SVN\}$, *and for each positive integers* $k \geq 2$ and $r \geq 1$,

$$\mathcal{L}[FV3\text{-}XSPkHFA(r)] \subsetneq \mathcal{L}[FV3\text{-}XSPkHFA(r{+}1)].$$

We next show a strong separation between self-verifying nondeterminism and nondeterminism.

Theorem 4.2. *For each positive integer* $k \geq 2$,

$$\mathcal{L}[FV3\text{-}NSPkHFA(2)] - \mathcal{L}[FV3\text{-}SVNSPkHFA] \neq \phi.$$

Proof : For each positive integer $k \geq 2$, let $T_4(k) = \{x \in \{0, 1\}^{(3)} \mid \exists n \geq \max\{4, k\}\ [l_1(x) = l_2(x) = l_3(x) = n] \wedge \exists i (1 \leq i \leq 2)$ [(the ith plane of x has exactly k '1's) $\wedge\ x[*, *, i] \neq x[*, *, i+2]]\}$. Then, by using the standard technique in [6], we can show that

$$T_4(2k\text{-}1) \in \mathcal{L}[FV3\text{-}NSPkHFA(2)] - \mathcal{L}[FV3\text{-}SVNSPkHFA].$$

From Theorems 4.1 and 4.2, we have the following corollary :

Corollary 4.2. *For each positive integers* $k \geq 2$ *and* $r \geq 2$,

(1) $\mathcal{L}[FV3\text{-}SVNSPkHFA] \subsetneq \mathcal{L}[FV3\text{-}NSPkHFA]$,

(2) $\mathcal{L}[FV3\text{-}SVNSPkHFA(r)] \subsetneq \mathcal{L}[FV3\text{-}NSPkHFA(r)]$,

and

(3) $\mathcal{L}[FV3\text{-}SVNSPkHFA(r{+}1)]$ *and* $\mathcal{L}[FV3\text{-}NSPkHFA(r)]$ *are incomparable.*

5 Conclusion

In this paper, we investigated path-bounded five-way three-dimensional finite automata, and showed some properties about them. It is interesting to investigate a hierarchy based on the degree of nondeterminism for six-way three-dimensional multihead finite automata which can move east, west, south, north, up, or down.

References

[1] J.Diaz and J.Toran, Classes of bounded nondeterminism, *Mathematical Systems Theory*, Vol.23, pp.21–32, 1990.

[2] P.Duris, J.Hromkovič, and K.Inoue, A separation of determinism, Las Vegas and nondeterminism for picture recognition, *To appear in JCSS.*

[3] J.Hromkovič, J.Karhumäki, H.Klauck, G.Schnitger, and S.Seibert, Measures of nondeterminism in finite automata, *To appear in JCSS.*

[4] J.Hromkovič, and G.Schnitger, Nondeterministic communication with a limited number of advice bits, *Proc. STOC* '96, *ACM*, pp.551–560, 1996.

[5] K.Inoue, I.Takanami, and H.Taniguchi, Three-way two-dimensional simple multihead finite automata – Hierarchical properties – *Trans. IECE* '79/2, Vol.62, No.2, pp.65–72, 1979.

[6] S.Inoue, A study of Path-bounded multihead finite automata, *Master Graduate School of Yamaguchi University*, 2003.

[7] S.Inoue, K.Inoue, A.Ito, Y.Wang, Path-bounded three-way two-dimensional finite automata, *in*, 2003 *Chugoku-Section Joint Convention Record of Institute of Electrical and Information Engineerings*, *Japan*, pp.262–263, 2003.

[8] M.Sakamoto, T.Ito, N.Tomozoe, and H.Furutani, A survey of three-dimensional automata, *The papers of technical meeting on information systems*, *IEE*, *Japan*, IS–07–12, pp.11–17, 2007.

[9] H.Taniguchi, K.Inoue, and I.Takanami, A note on three-dimensional finite automata, *Information Sciences*, Vol.26, pp.65–85, 1982.

[10] A.C.Yao and R.L.Rivest, k+1 heads are better than k, *J.ACM*, Vol.25, No.2, pp.337–340, 1978.